

Simply Gödel

Gödel's incompleteness theorems

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Gödel's incompleteness theorems are two theorems of mathematical logic that are concerned with the limits of provability in formal axiomatic theories. These results, published by Kurt Gödel in 1931, are important both in mathematical logic and in philosophy of mathematics. The theorems are interpreted as showing that Hilbert's program to find a complete and consistent set of axioms for all mathematics is impossible.

The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an effective procedure (i.e. an algorithm) is capable of proving all truths about the arithmetic of natural numbers. For any such consistent formal system, there will always be statements about natural numbers that are true, but that are unprovable within the system. Equivalently, there will always be statements about natural numbers that are false, but that cannot be proved false within the system.

The second incompleteness theorem, an extension of the first, shows that no such system can demonstrate its own consistency.

Employing a diagonal argument, Gödel's incompleteness theorems were among the first of several closely related theorems on the limitations of formal...

Gödel metric

The Gödel metric, also known as the Gödel solution or Gödel universe, is an exact solution, found in 1949 by Kurt Gödel, of the Einstein field equations

The Gödel metric, also known as the Gödel solution or Gödel universe, is an exact solution, found in 1949 by Kurt Gödel, of the Einstein field equations in which the stress-energy tensor contains two terms: the first representing the matter density of a homogeneous distribution of swirling dust particles (see Dust solution), and the second associated with a negative cosmological constant (see Λ -vacuum solution).

This solution has many unusual properties—in particular, the existence of closed time-like curves that would allow time travel in a universe described by the solution. Its definition is somewhat artificial, since the value of the cosmological constant must be carefully chosen to correspond to the density of the dust grains, but this spacetime is an important pedagogical example.

== Definition ==

Like any other Lorentzian spacetime, the Gödel solution represents the metric tensor in terms of a local coordinate chart. It may be easiest to understand the Gödel universe using the cylindrical coordinate system (see below), but this article uses the chart originally used by Gödel. In this chart, the metric (or, equivalently, the line element) is

Gödel's ontological proof

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Gödel's ontological proof is a formal argument by the mathematician Kurt Gödel (1906–1978) for the existence of God. The argument is in a line of development that goes back to Anselm of Canterbury (1033–1109). St. Anselm's ontological argument, in its most succinct form, is as follows: God, if He does exist, is that for which no greater can be conceived. God exists in the understanding. If God exists in the understanding, we could imagine Him to be greater by existing in reality. Therefore, God must exist. A more elaborate version was given by Gottfried Leibniz (1646–1716); this is the version that Gödel studied and attempted to clarify with his ontological argument.

The argument uses modal logic, which deals with statements about what is necessarily true or possibly true. From the axioms that a property can only be positive if not-having-it is not positive, and that properties implied by a positive property must all also be themselves positive, it concludes that (since positive properties do not involve contradiction) for any positive property, there is possibly a being that instantiates it. It defines God as the being instantiating all positive properties. After defining what it means...

Gödel's speed-up theorem

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In mathematics, Gödel's speed-up theorem, proved by Gödel (1936), shows that there are theorems whose proofs can be drastically shortened by working in more powerful axiomatic systems.

Kurt Gödel showed how to find explicit examples of statements in formal systems that are provable in that system but whose shortest proof is unimaginably long. For example, the statement:

"This statement cannot be proved in Peano arithmetic in fewer than a googolplex symbols"

is provable in Peano arithmetic (PA) but the shortest proof has at least a googolplex symbols, by an argument similar to the proof of Gödel's first incompleteness theorem: If PA

is consistent, then it cannot prove the statement in fewer than a googolplex symbols, because the existence of such a proof would itself be a theorem of PA, a contradiction. But simply enumerating all strings of length up to a googolplex and checking that each such string is not a proof (in PA) of the statement, yields a proof of the statement (which is necessarily longer than a googolplex symbols).

The statement has a short proof in a more powerful system: in fact the proof given in the previous paragraph is a proof in the system of Peano arithmetic plus...

Proof sketch for Gödel's first incompleteness theorem

This article gives a sketch of a proof of the first of Gödel's incompleteness theorems. This theorem applies to any formal theory that satisfies certain

This article gives a sketch of a proof of the first of Gödel's incompleteness theorems. This theorem applies to any formal theory that satisfies certain technical hypotheses, which are discussed as needed during the sketch. We will assume for the remainder of the article that a fixed theory satisfying these hypotheses has been selected.

Throughout this article the word "number" refers to a natural number (including 0). The key property these numbers possess is that any natural number can be obtained by starting with the number 0 and adding 1 a finite number of times.

== Hypotheses of the theory ==

Gödel's theorem applies to any formal theory that satisfies certain properties. Each formal theory has a signature that specifies the nonlogical symbols in the language of the theory. For simplicity, we will assume that the language of the theory is composed from the following collection of 15 (and only 15) symbols:

Gödel numbering for sequences

In mathematics, a Gödel numbering for sequences provides an effective way to represent each finite sequence of natural numbers as a single natural number

In mathematics, a Gödel numbering for sequences provides an effective way to represent each finite sequence of natural numbers as a single natural number. While a set theoretical embedding is surely possible, the emphasis is on the effectiveness of the functions manipulating such representations of sequences: the operations on sequences (accessing individual members, concatenation) can be "implemented" using total recursive functions, and in fact by primitive recursive functions.

It is usually used to build sequential "data types" in arithmetic-based formalizations of some fundamental notions of mathematics. It is a specific case of the more general idea of Gödel numbering. For example, recursive function theory can be regarded as a formalization of

the notion of an algorithm, and can be regarded as a programming language to mimic lists by encoding a sequence of natural numbers in a single natural number.

== Gödel numbering ==

Godel (river)

Godel is a river of the North Frisian island Föhr, Schleswig-Holstein, Germany. The Godel springs at Witsum and Utersum from several tidal creeks. It

Godel is a river of the North Frisian island Föhr, Schleswig-Holstein, Germany.

The Godel springs at Witsum and Utersum from several tidal creeks. It discharges southeast of Witsum into the North Sea.

== Gallery ==

Original proof of Gödel's completeness theorem

The proof of Gödel's completeness theorem given by Kurt Gödel in his doctoral dissertation of 1929 (and a shorter version of the proof, published as an

The proof of Gödel's completeness theorem given by Kurt Gödel in his doctoral dissertation of 1929 (and a shorter version of the proof, published as an article in 1930, titled "The completeness of the axioms of the functional calculus of logic" (in German)) is not easy to read today; it uses concepts and formalisms that are no longer used and terminology that is often obscure. The version given below attempts to represent all the steps in the proof and all the important ideas faithfully, while restating the proof in the modern language of mathematical logic. This outline should not be considered a rigorous proof of the theorem.

== Assumptions ==

We work with first-order predicate calculus. Our languages allow constant, function and relation symbols. Structures consist of (non-empty) domains and interpretations of the relevant symbols as constant members, functions or relations over that domain.

We assume classical logic (as opposed to intuitionistic logic for example).

We fix some axiomatization (i.e. a syntax-based, machine-manageable proof system) of the predicate calculus: logical axioms and rules of inference. Any of the several well-known equivalent axiomatizations will do. Gödel...

Slingshot argument

controversially, that the most compelling version was suggested by Kurt Gödel (1944). These arguments are sometimes modified to support the alternative

In philosophical logic, a slingshot argument is one of a group of arguments claiming to show that all true sentences stand for the same thing.

This type of argument was dubbed the "slingshot" by philosophers Jon Barwise and John Perry (1981) due to its disarming simplicity. It is usually said that versions of the slingshot argument have been given by Gottlob Frege, Alonzo Church, W. V. Quine, and Donald Davidson. However, it has been disputed by Lorenz Krüger (1995) that there is much unity in this tradition. Moreover, Krüger rejects Davidson's claim that the argument can refute the correspondence theory of truth. Stephen Neale (1995) claims, controversially, that the most compelling version was suggested by Kurt Gödel (1944).

These arguments are sometimes modified to support the alternative, and evidently stronger, conclusion that there is only one fact, or one true proposition, state of affairs, truth condition, truthmaker, and so on.

== The argument ==

One version of the argument (Perry 1996) proceeds as follows.

Assumptions:

Hans Hahn (mathematician)

Kultur: Band 6: Te-Z, p. 403, Springer-Verlag, 2017 Richard Tieszen, Simply Gödel, Simply Charly, New York, 2017. Josephson-Storm, Jason (2017). The Myth of

Hans Hahn (; German: [ha:n]; 27 September 1879 – 24 July 1934) was an Austrian mathematician and philosopher who made contributions to functional analysis, topology, set theory, the calculus of variations, real analysis, and order theory. In philosophy he was among the main logical positivists of the Vienna Circle.

== Biography ==

Born in Vienna as the son of a higher government official of the K.K. Telegraphen-Korrespondenz Bureau, in 1898 Hahn became a student at the Universität Wien starting with a study of law. In 1899 he switched over to mathematics and spent some time at the universities of Strasbourg, Munich and Göttingen. In 1902 he took his Ph.D. in Vienna, on the subject "Zur Theorie der zweiten Variation einfacher Integrale". He was a student of Gustav von Escherich.

He was appointed to the teaching staff (habilitation) in Vienna in 1905. After 1905/1906 as a stand-in for Otto Stolz at the University of Innsbruck and some further years as a Privatdozent in Vienna, he was nominated in 1909 Professor extraordinarius in Czernowitz, at that time a town within the empire of Austria. After joining the Austrian army in 1915, he was badly wounded in 1916 and became again Professor...

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