Chapter 6 Random Variables Continuous Case

Probability distribution

many different random values. Probability distributions can be defined in different ways and for discrete or for continuous variables. Distributions with

In probability theory and statistics, a probability distribution is a function that gives the probabilities of occurrence of possible events for an experiment. It is a mathematical description of a random phenomenon in terms of its sample space and the probabilities of events (subsets of the sample space).

For instance, if X is used to denote the outcome of a coin toss ("the experiment"), then the probability distribution of X would take the value 0.5 (1 in 2 or 1/2) for X = heads, and 0.5 for X = tails (assuming that the coin is fair). More commonly, probability distributions are used to compare the relative occurrence of many different random values.

Probability distributions can be defined in different ways and for discrete or for continuous variables. Distributions with special properties...

Exchangeable random variables

identically distributed random variables in statistical models. Exchangeable sequences of random variables arise in cases of simple random sampling. Formally

In statistics, an exchangeable sequence of random variables (also sometimes interchangeable) is a sequence X1, X2, X3, ... (which may be finitely or infinitely long) whose joint probability distribution does not change when the positions in the sequence in which finitely many of them appear are altered. In other words, the joint distribution is invariant to finite permutation. Thus, for example the sequences

X

1

,

X

2

,

X

3

,

X

4

,

X

5...

Convergence of random variables

there exist several different notions of convergence of sequences of random variables, including convergence in probability, convergence in distribution

In probability theory, there exist several different notions of convergence of sequences of random variables, including convergence in probability, convergence in distribution, and almost sure convergence. The different notions of convergence capture different properties about the sequence, with some notions of convergence being stronger than others. For example, convergence in distribution tells us about the limit distribution of a sequence of random variables. This is a weaker notion than convergence in probability, which tells us about the value a random variable will take, rather than just the distribution.

The concept is important in probability theory, and its applications to statistics and stochastic processes. The same concepts are known in more general mathematics as stochastic convergence...

Probability theory

Central subjects in probability theory include discrete and continuous random variables, probability distributions, and stochastic processes (which provide

Probability theory or probability calculus is the branch of mathematics concerned with probability. Although there are several different probability interpretations, probability theory treats the concept in a rigorous mathematical manner by expressing it through a set of axioms. Typically these axioms formalise probability in terms of a probability space, which assigns a measure taking values between 0 and 1, termed the probability measure, to a set of outcomes called the sample space. Any specified subset of the sample space is called an event.

Central subjects in probability theory include discrete and continuous random variables, probability distributions, and stochastic processes (which provide mathematical abstractions of non-deterministic or uncertain processes or measured quantities...

Random walk

walk formally, take independent random variables Z 1 , Z 2 , ... {\displaystyle Z_{1},Z_{2},\dots } , where each variable is either 1 or −1, with a 50% probability

In mathematics, a random walk, sometimes known as a drunkard's walk, is a stochastic process that describes a path that consists of a succession of random steps on some mathematical space.

An elementary example of a random walk is the random walk on the integer number line

Z

{\displaystyle \mathbb {Z} }

which starts at 0, and at each step moves +1 or −1 with equal probability. Other examples include the path traced by a molecule as it travels in a liquid or a gas (see Brownian motion), the search path of a foraging animal, or the price of a fluctuating stock and the financial status of a gambler. Random walks have applications to engineering and many scientific fields including ecology, psychology, computer science, physics, chemistry...

Uncorrelatedness (probability theory)

orthogonality, except in the special case where at least one of the two random variables has an expected value of 0. In this case, the covariance is the expectation

In probability theory and statistics, two real-valued random variables,

X

{\displaystyle X}

,

Y

{\displaystyle Y}

, are said to be uncorrelated if their covariance,

cov

[

X

,

Y

]

=

E

[

X

Y

]

−

E

[

X

]

E

[

Y

]

{\displaystyle \operatorname {cov} [X,Y]=\operatorname {E} [XY]-\operatorname {E} [X]\operatorname {E} [Y]}

, is zero. If two variables are uncorrelated, there is no linear relationship between them.

Uncorrelated random variables have a Pearson correlation coefficient...

Probability density function

discrete random variables (random variables that take values on a countable set), while the PDF is used in the context of continuous random variables. Suppose

In probability theory, a probability density function (PDF), density function, or density of an absolutely continuous random variable, is a function whose value at any given sample (or point) in the sample space (the set of possible values taken by the random variable) can be interpreted as providing a relative likelihood that the value of the random variable would be equal to that sample. Probability density is the probability per unit length, in other words. While the absolute likelihood for a continuous random variable to take on any particular value is zero, given there is an infinite set of possible values to begin with. Therefore, the value of the PDF at two different samples can be used to infer, in any particular draw of the random variable, how much more likely it is that the random...

Continuous-time Markov chain

changing state according to the least value of a set of exponential random variables, one for each possible state it can move to, with the parameters determined

A continuous-time Markov chain (CTMC) is a continuous stochastic process in which, for each state, the process will change state according to an exponential random variable and then move to a different state as specified by the probabilities of a stochastic matrix. An equivalent formulation describes the process as changing state according to the least value of a set of exponential random variables, one for each possible state it can move to, with the parameters determined by the current state.

An example of a CTMC with three states

{

0

,

1

,

2

}

{\displaystyle \{0,1,2\}}

is as follows: the process makes a transition after the amount of time specified by the holding time—an exponential random variable...

Errors-in-variables model

errors-in-variables model or a measurement error model is a regression model that accounts for measurement errors in the independent variables. In contrast

In statistics, an errors-in-variables model or a measurement error model is a regression model that accounts for measurement errors in the independent variables. In contrast, standard regression models assume that those regressors have been measured exactly, or observed without error; as such, those models account only for errors in the dependent variables, or responses.

In the case when some regressors have been measured with errors, estimation based on the standard assumption leads to inconsistent estimates, meaning that the parameter estimates do not tend to the true values even in very large samples. For simple linear regression the effect is an underestimate of the coefficient, known as the attenuation bias. In non-linear models the direction of the bias is likely to be more complicated...

Expected value

dx} for any absolutely continuous random variable X. The above discussion of continuous random variables is thus a special case of the general Lebesgue

In probability theory, the expected value (also called expectation, expectancy, expectation operator, mathematical expectation, mean, expectation value, or first moment) is a generalization of the weighted average. Informally, the expected value is the mean of the possible values a random variable can take, weighted by the probability of those outcomes. Since it is obtained through arithmetic, the expected value sometimes may not even be included in the sample data set; it is not the value you would expect to get in reality.

The expected value of a random variable with a finite number of outcomes is a weighted average of all possible outcomes. In the case of a continuum of possible outcomes, the expectation is defined by integration. In the axiomatic foundation for probability provided by measure...